Modeling Route Duration in Mobile Ad-Hoc Networks

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Abstract

An analytical model to estimate the time duration of routes formed by several intermediate nodes in MANETs is presented. Although there are works related to estimation of route duration, they only partially obtained analytical expressions. First, we approach the route duration problem by modeling a 3-node static case, source and destination (static) and one intermediate node (mobile). Then, a 3-node mobile case is analyzed. For both cases, we evaluate how long the route is valid. Finally, a K-node mobile case is presented. We conclude that the time duration of a route formed by N intermediate nodes can be accurately found from the minimum time duration of N separated 3-node routes. Simulations were developed using NS-2 to verify the proposed model. This work can be used to compute the signaling overhead during route-maintenance of routing protocols for MANETs and to adjust the maximum network size.

Keywords: ad-hoc networks, route duration.

1 Introduction

An ad-hoc network is a collection of nodes forming a temporary network by means of wireless interfaces and without use of any existing network infrastructure or centralized administration.

In Mobile Ad-hoc Networks (MANETs), network nodes are free to move randomly and self-organize; thus, the network topology may change rapidly and unpredictably. Such a network may operate in a standalone fashion, or may be connected to the Internet. MANETs became a popular subject for research as laptops and 802.11/Wi-Fi wireless networking were widespread in the middle to late 1990s. Other types of ad-hoc networks that are becoming increasingly popular are the Vehicular Ad-hoc Networks (VANETs) and Wireless Sensor Networks (WSNs).

Degree of mobility is an important factor and a key research issue in MANET and VANET networks. Up to a large extent, degree of mobility determines the time duration of routes, so an analytical study about route duration turns out to be crucial. Although, at this moment, most sensor applications have zero or low mobility, it can be anticipated that future sensor network applications would involve some degree of mobility [5]. The analytical study presented in this paper would be an important contribution for this kind of sensor application. A study of route duration may not be easily tractable even under simple mobility models, however a route duration model would be helpful to anticipate route disruption and to avoid the degradation of system performance. Previous knowledge of route duration can be used to select an alternative route before the current one fails, and it can also be used to decrease or limit packet losses and latency due to signaling overhead during route reconstruction. Because route duration decreases with route length, a route duration model could be used to scale up/down the maximum network size as to meet minimum route duration requirements to guarantee a satisfactory communication between any pair of nodes.

Due to the limited transmission range of wireless radio transceivers, multiple nodes working as relays (multi-hop routing) may be needed for one node to exchange data with another node across the ad-hoc network. Traffic relaying in mobile ad-hoc networks, however, is a difficult task to deal with. Node mobility, signal interference and power outages make the network conditions change frequently. As a consequence, any link along a route may fail at some point, and every time a route fails, the routing protocol needs to search for another route. It is clear that route duration strongly depends on the mobility pattern of the nodes, and one would like to be able to compute this duration in advance.

In general, there will be a need for none, one, or several intermediate forwarding nodes between source-destination pairs in an ad-hoc network depending on the distance between source and destination nodes, the transmission range and node density. As shown in Figure 1, each intermediate

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forwarding node must be located within an *oval-like* overlapping region formed by the intersection of the coverage zones between its adjacent route neighbors.

The main contribution of this paper is to present an analytical study which predicts the time duration of routes in a mobile ad-hoc network. Previous researchers have analyzed this problem, but they have not truly achieved the goal of obtaining an expression for route duration. We believe an analytical study is more powerful and has a higher applicability than a simulation or empirical model for example, because it is not tied to any specific simulation scenario. The rest of the paper is as follows: Section II presents a description of previous work in this area. A study of route duration of a 3-node route is presented in Section III for the case where only the intermediate node moves, and Section IV presents the case where all three nodes move. In Section V we generalize the model to routes with N intermediate nodes. Section VI presents simulations using NS -2 network simulator to compare the analytical model versus simulation results. Finally, in Section VII we present some final remarks.

2 Related Work

Simulation has been one of the main methods for analyzing the properties of route duration in mobile ad-hoc networks in the past. Simulations have considered several parameters such as mobility model, traffic pattern, propagation model, etc. The authors in [3] made one of the first studies concerning the analysis of route duration based on empirical results obtained by simulations. These authors examined detailed statistics of route duration considering the following mobility models: Random Way Point (RWP) [4], Reference Point Group Mobility (RPGM) [12], Freeway (FW) [2] and Manhattan (MH) [2]. They observed that, under certain conditions (i.e., a minimum speed and routes with several hops), the time duration of routes can be approximated by exponential distributions. They evaluated the effect of the number of hops, the transmission range and the relative speed of the mobility model on route duration. However, the authors did not consider the goodness of fit of any other distribution. Moreover, they did not justify the selection of an exponential distribution with any mathematical validation. To cope with this limitation, the authors in [11] used the Palm's theorem to state that, under some circumstances (e.g., infinite node density), the lifetime associated to routes with a large number of hops converges to an exponential distribution. These works provide a solution for the analysis of paths which is valid only for routes with a large number of hops. Therefore, their study could not be applied to many practical MANET applications where the paths consist of few hops only. In spite of these limitations, the popularity of the exponential fitting has been used as a common approximation in some other works such as in [1]. In [1], the authors presented a statistical model for estimating route expiry time adaptively, to reduce control traffic in on-demand ad-hoc networks.

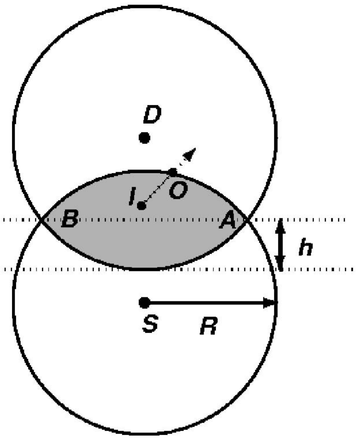

Figure 1. Overlapping region of two adjacent nodes

There are several analytical studies related to route duration in the literature. Even though the authors of these works followed different approaches to solve the route duration problem, they did not truly achieved the goal of obtaining an expression for modeling duration of routes with several intermediate nodes, which limit their applicability. For example, the authors in [6] presented a simplified model of link duration for a single-hop case. Based on this model, they tried to generalize a model for a multi-hop route, but they did not provide any closed-form solution for this model. In [10], the authors presented an analysis of link duration for a two-hop MANET only. In this study, the authors considered an exponential distribution of route duration and they assumed that the source and destination nodes are fixed while the intermediate node is moving using the RWP mobility model. However, they did not extend their analysis to a route of several hops. The authors in [14] assumed that link durations are independent and exponentially distributed random variables with a known value for the mean link duration. Based on these assumptions, the authors derived some expressions to estimate route duration for single and multiple routes. However, in most cases, it cannot be assumed that the mean link value is a known parameter. In [16], the authors presented a framework for studying route duration in mobile ad-hoc networks based on various mobility models, but they did not present any detailed analytical expressions. In [15], the authors derived the joint probability distribution of route duration, but only for the case of a discrete-time, using the random walk model. They based their analysis of route duration partitioning the MANET network into a number of hexagonal cells and assuming that mobile nodes roam around in a cell-to-cell basis. In [17], the authors described the probability distribution function of route duration assuming that nodes move according to a constant velocity model and derived the statistic results of link and route duration in ad-hoc network. We decided that the analytical works presented in [15] and [17] are the closest models to ours. In Section VI, we will present a comparison between these models and ours.

The mobility model proposed in [9] used fluid-flow techniques to analytically model the average sojourn time of an intermediate node while it crosses the region formed by the intersection of the coverage zones between its adjacent nodes (overlapping region). This model considered that intermediate nodes are found right after entering the overlapping region. But they did not reflect on the possibility that the forwarding node is already located within this region, which is the usual case, so route duration estimated by this model would be much different than the real value. Although this model assumed various intermediate nodes, because it considered all overlapping regions have similar size, the actual sojourn time for each forwarding node in the route would be the same, thus the route duration predicted by this model will be the same for routes with one or several intermediate nodes, which is not realistic.

Although a study of route duration is extremely difficult to deal with even under simple mobility patterns, in this paper we propose a route duration model aimed to remove some of the limitations found in previous works. It is important to stand out that does not exist a general mobility model that contemplates all possible dynamic behaviors that a node in wireless networks can follow. Our model predicts the time duration of routes with any number of intermediate nodes, and the only assumption we have made is that all nodes move following a constant velocity model, (i.e., from the route discovery time up to the route failure time, all nodes involved in the route are moving at constant speeds and follow random rectilinear trajectories). In future work we plan to study the behavior of our route duration model with other mobility models.

3 Route Analysis (3-node Static Case)

We approach the route duration problem by modeling and understanding simpler route scenarios first, before moving to more complex ones. First, a route formed by three nodes will be modeled, considering that only the intermediate forwarding node is moving while source and destination nodes remain static. Then, a route formed by three mobile nodes will be analyzed. Finally, a general case of a route formed by several mobile nodes will be presented.

In a real route, intermediate nodes will be found inside the overlapping regions. In addition, the size of the overlapping region changes for each intermediate node, thus the sojourn time of a forwarding node within this region could vary significantly for different intermediate nodes. To include these considerations in our analysis, it was necessary to involve all possible initial positions and trajectories of the nodes in the route and consider different sizes of the overlapping region.

First, let the source and destination nodes be fixed at points $S(x_S, y_S)$ and $D(x_D, y_D)$, respectively (see Figure 1), and let their transmission range be R meters, so the coverage zone of each node has the shape of a circle with radius R. As illustrated in Figure 1, factor h is an indicator of how much area overlaps between the adjacent nodes of each forwarding node, this factor plays a crucial role in the operation and performance of routing protocols for wireless ad-hoc networks because it represents the size of the overlapping region (represented by shaded area in Figure 1). Furthermore, let points $A(x_A, y_A)$ and $B(x_B, y_B)$ be the intersection points between both circles. It is easy to show that the coordinates of points A and B, as well as factor h can be found by:

$$x_{A/B} = \frac{-B' \pm \sqrt{B'^2 - 4A'C'}}{2A'}$$

$$y_{A/B} = \frac{2(x_D - x_S)x_{A/B} + (x_S^2 - x_D^2) + (y_S^2 - y_D^2)}{2(y_S - y_D)}$$

$$h = R - \frac{d_{S-D}}{2}$$

where:

$$A' = 4[(x_S - x_D)^2 + (y_S - y_D)^2]$$

$$B' = -4[(x_S^2 - x_D^2)(x_S - x_D) + (y_S - y_D)^2(x_S + x_D)]$$

$$C' = [(x_S^2 - x_D^2) - (y_S - y_D)^2]^2 + 4(x_S^2 - R^2)(y_S - y_D)^2$$

$$d_{S-D} = \sqrt{(x_S - x_D)^2 - (y_S - y_D)^2}$$

Now, let point $I(x_I, y_I)$ be the initial position of the intermediate node and point $O(x_O, y_O)$ be the position where this node leaves the overlapping region while following a straight trajectory sloped α_I degrees and moving at constant speed v_I. The distance travelled by an intermediate node moving from I to O (d_{I-O}) can be found by geometric analysis as:

$$d_{I-O}(\alpha_I) = |-\sqrt{a^2 + b^2}\sin(\alpha_I + \delta) + \\ + \sqrt{R^2 - [(a^2 + b^2)\cos^2(\alpha_I + \delta)]}| \tag{1}$$

Parameters a and b, found in the equations of this section, must be computed separately to analyze the link between source - intermediate node and the link between intermediate node - destination. When the intermediate node

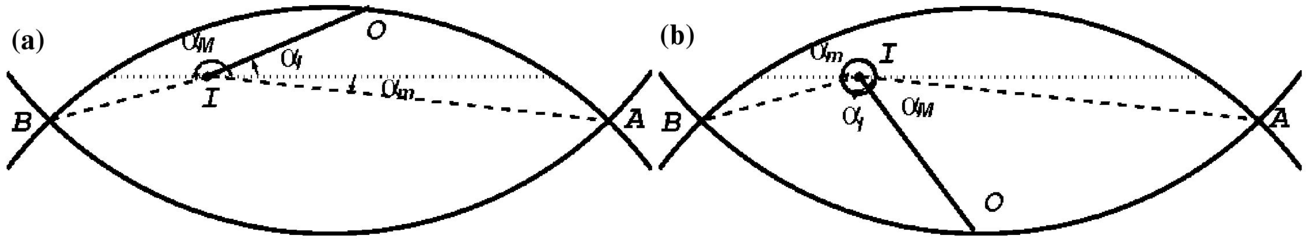

Figure 2. (a) Angles involved while modeling link duration of source - intermediate node. (b) Angles involved while modeling link duration of intermediate node - destination

crosses over the border of the source node coverage zone, parameters a and b are:

$$a = y_I - y_S \quad ; \quad b = x_I - x_S \quad ; \quad \delta = \arctan\left(\tfrac{b}{a}\right)$$

otherwise

$$a = y_I - y_D \quad ; \quad b = x_I - x_D \quad ; \quad \delta = \arctan\left(\tfrac{b}{a}\right)$$

Then, the average distance travelled by an intermediate node before leaving the overlapping region ($\overline{d}_{I-O}$), given its initial position $I(x_I, y_I)$, can be found using the Mean Value Theorem [13], this is:

$$\overline{d}_{I-O} = \frac{1}{\Delta\alpha_I}\int_{\alpha_m}^{\alpha_M} d_{I-O}\left(\alpha_I\right) d\alpha_I \qquad (2)$$

where:

$$\Delta\alpha_I = |\alpha_M - \alpha_m|$$

$$\alpha_M = \arctan\left(\tfrac{y_B - y_I}{x_B - x_I}\right) \quad ; \quad \alpha_m = \arctan\left(\tfrac{y_A - y_I}{x_A - x_I}\right)$$

and

$$\alpha_M = \arctan\left(\tfrac{y_A - y_I}{x_A - x_I}\right) \quad ; \quad \alpha_m = \arctan\left(\tfrac{y_B - y_I}{x_B - x_I}\right)$$

Therefore,

$$\overline{d}_{I-O} = \frac{1}{\Delta\alpha_I}[\sqrt{a^2+b^2}\cos\left(\alpha_I+\delta\right)|_{\alpha_I=\alpha_m}^{\alpha_I=\alpha_M} +$$

$$+\int_{\alpha_m}^{\alpha_M}\sqrt{R^2-[(a^2+b^2)\cos^2\left(\alpha_I+\delta\right)]}d\alpha_I] \qquad (3)$$

As well as parameters a and b, the angles δ, α_m and α_M, found in the equations of this section, must be computed separately for the link between source - intermediate node and for the link between intermediate node - destination. In this analysis, the angle α_I is an independent random variable uniformly distributed over the interval ($\Delta\alpha_I$) given by the difference between its maximum and minimum values (α_M and α_m, respectively). Figures 2a and 2b show the angles involved for the links between source - intermediate node and intermediate node - destination, respectively.

Because Eq. (3) does not seem to be possible to integrate it algebraically, we found an approximate integration by replacing the square root in the integrand with a binomial series [13] and, after some algebra, it can be simplified as shown:

$$\sqrt{R^2-[(a^2+b^2)\cos^2\left(\alpha_I+\delta\right)]} \approx$$

$$R\left(1-\sum_{j=1}^{\infty}k_j\left(u\left(\alpha_I\right)\right)^j\right)$$

where:

$$u\left(\alpha_I\right) = \cos^2\left(\alpha_I+\delta\right)$$

$$k_j = \frac{1}{(j)!}\left(\frac{(a^2+b^2)}{R^2}\right)^j\left[\prod_{l=1}^{j}\left(\frac{3}{2}-l\right)\right]$$

then:

$$R\int_{\alpha_m}^{\alpha_M}\left(1-\sum_{j=1}^{\infty}k_j\left(u\left(\alpha_I\right)\right)^j\right)d\alpha_I =$$

$$R\left(\alpha_I|_{\alpha_I=\alpha_m}^{\alpha_I=\alpha_M}-\sum_{j=1}^{\infty}k_j\int_{\alpha_m}^{\alpha_M}\cos^{2j}\left(\alpha_I+\delta\right)d\alpha_I\right) \qquad (4)$$

After replacing the integral found in (3) by (4), an approximation of the average distance will be:

$$\overline{d}_{I-O} \approx \frac{1}{\Delta\alpha_I}[\sqrt{a^2+b^2}\cos\left(\alpha_I+\delta\right)|_{\alpha_I=\alpha_m}^{\alpha_I=\alpha_M} +$$

$$+R\left(\alpha_I|_{\alpha_I=\alpha_m}^{\alpha_I=\alpha_M}-\sum_{j=1}^{\infty}k_j\int_{\alpha_m}^{\alpha_M}\cos^{2j}\left(\alpha_I+\delta\right)d\alpha_I\right)] \qquad (5)$$

In order to remove dependence on the initial position $I(x_I, y_I)$, the average distance over all possible initial positions within the overlapping region, may be computed by means of:

$$\overline{D}_{I-O} = \int_{\alpha_m}^{\alpha_M} \int_{y_m}^{y_M} \int_{x_m}^{x_M} f_{\alpha_I x_I y_I}(\alpha_I, x_I, y_I)$$

$$d_{I-O}(x_I, y_I, \alpha_I)\, dx_I dy_I d\alpha_I \qquad (6)$$

where:

$f_{\alpha_I x_I y_I}(\alpha_I, x_I, y_I)$ represents the joint probability density function for the random variables α_I, x_I and y_I .

The average time that a forwarding node remains inside the overlapping region (T_I) can be found dividing the average distance travelled by a mobile intermediate node ($\overline{d}_{I-O}$) by its speed of movement (v_I), therefore:

$$T_I = \frac{\overline{d}_{I-O}}{v_I} \qquad (7)$$

4 Route Analysis (3-Node Mobile Case)

Now, we analyze 3-node routes where source, destination and forwarding nodes are all mobile. We again model how long it takes for the intermediate node to exit the overlapping region. In this case, we consider all nodes are moving with random rectilinear trajectories at constant speeds. Let us identify the source, intermediate and destination nodes with indexes S, I and D, respectively. Let us denote with k such index, thus $k = S, I$ or D. Each node's position is described by the coordinates $(x_k(t), y_k(t))$. Let $\vec{v}_k(t)$ be the velocity vector of node k, this is:

$$\vec{v}_k(t) = [v_{x_k}(t)]\,\hat{\imath} + [v_{y_k}(t)]\,\hat{\jmath} \qquad (8)$$

where: $\hat{\imath}, \hat{\jmath}$: unit coordinated vectors.

If each node k is following a straight trajectory sloped at a α_k degrees and it is moving at a constant speed v_k. Then, its velocity vector ($\vec{v}_k(t)$), would be given by:

$$\vec{v}_k(t) = [v_k \cos(\alpha_k)]\,\hat{\imath} + [v_k \sin(\alpha_k)]\,\hat{\jmath} \qquad (9)$$

$$|\vec{v}_k(t)| = v_k \qquad [m/s]$$

Now, let $\vec{r}_k(t)$ be the vector that describes the position of node k, that is:

$$\vec{r}_k(t) = \vec{r}_k(0) + \int_o^t \vec{v}_k(t)\, dt \qquad (10)$$

where:

$$\vec{r}_k(0) = x_k(0)\,\hat{\imath} + y_k(0)\,\hat{\jmath}$$

$\vec{r}_k(0)$: initial position vector of node k.

Substituting Equation (9) in (10), we get:

$$\vec{r}_k(t) = [v_k t \cos(\alpha_k) + x_k(0)]\,\hat{\imath} +$$

$$+ [v_k t \sin(\alpha_k) + y_k(0)]\,\hat{\jmath} \qquad (11)$$

$$\vec{r}_k(t) = [x_k(t)]\,\hat{\imath} + [y_k(t)]\,\hat{\jmath} \qquad (12)$$

where:

$$x_k(t) = v_k t \cos(\alpha_k) + x_k(0)$$

is the abscissa of position of node k and

$$y_k(t) = v_k t \sin(\alpha_k) + y_k(0)$$

is the ordinate of position of node k.

Now, let $d_{S-I}(t)$ be the distance between the source and intermediate nodes and let Let $d_{I-D}(t)$ be the distance between the intermediate and destination nodes. Distances $d_{S-I}(t)$ and $d_{I-D}(t)$ can be found by the Euclidean distance formula, so:

$$d_{S-I}(t) = |\vec{r}_I(t) - \vec{r}_S(t)| \qquad (13)$$

$$d_{I-D}(t) = |\vec{r}_D(t) - \vec{r}_I(t)| \qquad (14)$$

Then,

$$d_{S-I}(t) = \sqrt{[\lambda t + \mu]^2 + [\phi t + \psi]^2} \qquad (15)$$

$$d_{I-D}(t) = \sqrt{[\lambda' t + \mu']^2 + [\phi' t + \psi']^2} \qquad (16)$$

where:

$$\lambda = v_I \cos(\alpha_I) - v_S \cos(\alpha_S)$$

$$\mu = x_I(0) - x_S(0)$$

$$\phi = v_I \sin(\alpha_I) - v_S \sin(\alpha_S)$$

$$\psi = y_I(0) - y_S(0)$$

and

$$\lambda' = v_D \cos(\alpha_D) - v_I \cos(\alpha_I)$$

$$\mu' = x_D(0) - x_I(0)$$

$$\phi' = v_D \sin(\alpha_D) - v_I \sin(\alpha_I)$$

$$\psi' = y_D(0) - y_I(0)$$

When either distance $d_{S-I}(t)$ or $d_{I-D}(t)$ exceeds the transmission range (R), the communication between the respective adjacent node pair $[S, I]$ or $[I, D]$ is interrupted, this is:

$$d_{S-I}(t) \geq R \qquad (17)$$

$$d_{I-D}(t) \geq R \qquad (18)$$

Let $T_{[S,I]}$ and $T_{[I,D]}$ be the rupture time of communication between adjacent node pairs $[S, I]$ and $[I, D]$, respectively. The rupture times can be found by:

$$T_{[S,I]} = \frac{-\Psi \pm \sqrt{\Psi^2 - 4\Phi\Omega}}{2\Phi} \quad [s] \qquad (19)$$

$$T_{[I,D]} = \frac{-\Psi' \pm \sqrt{\Psi'^2 - 4\Phi'\Omega'}}{2\Phi'} \quad [s] \qquad (20)$$

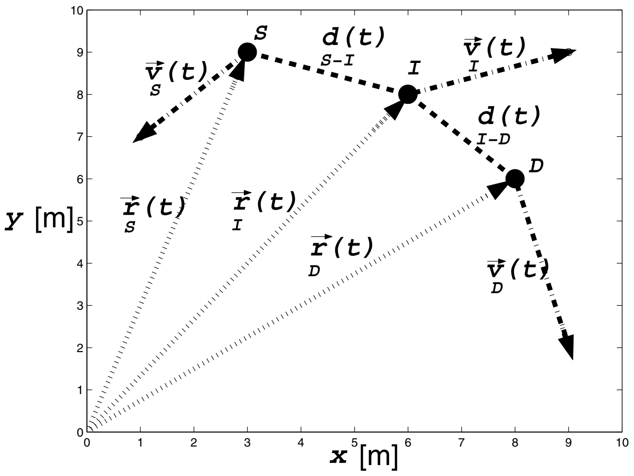

Figure 3. Position and velocity vectors for source, intermediate and destination nodes

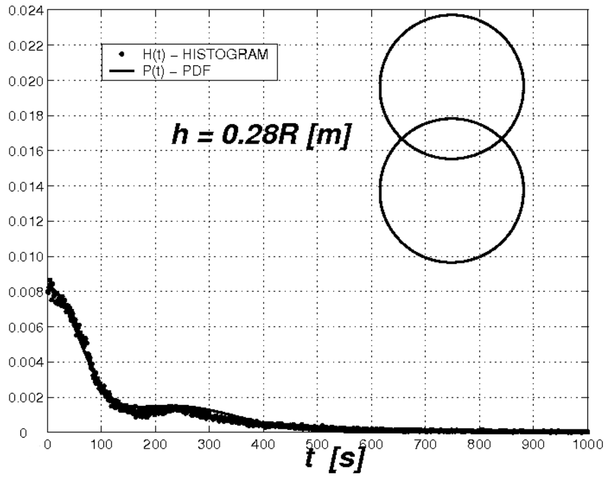

Figure 4. Probability Density Functions for $h = 0.28R\,[m]$

These are the solutions of the equations:

$$\Phi t^2 + \Psi t + \Omega = 0 \tag{21}$$

$$\Phi' t^2 + \Psi' t + \Omega' = 0 \tag{22}$$

where:

$$\Phi = \lambda^2 + \phi^2$$

$$\Psi = 2\left[\lambda\mu + \phi\psi\right]$$

$$\Omega = \mu^2 + \psi^2 - R^2$$

and

$$\Phi' = \lambda'^2 + \phi'^2$$

$$\Psi' = 2\left[\lambda'\mu' + \phi'\psi'\right]$$

$$\Omega' = \mu'^2 + \psi'^2 - R^2$$

Finally, the route duration (T_I) will be found by:

$$T_I = \min\left(T_{[S,I]}, T_{[I,D]}\right) \tag{23}$$

Figure 3 illustrates the position and velocity vectors of a route with 3 mobile nodes (source S, intermediate I and destination D). This figure shows the corresponding vector positions at instant t when each node is moving following the direction described by its velocity vector.

This case is far more complex to analyze than the 3-node static case because the size and location of the overlapping region are constantly changing as time passes and, consequently, the factor h varies in the same way. In order to get an average value of route duration for any route, it would be necessary to consider all possible trajectories and initial positions for the three nodes involved in the route. Figure 4 shows an histogram representing the relative frequency of the time duration of a set of routes. Route duration is given by the minimum time that each forwarding node remains inside of its respective overlapping region of the route using Equation 23, for an initial h value corresponding to an average overlapping region ($h = 0.28R\ [m]$). This h value has been obtained by means of an exhaustive analysis of 10,000 routes formed by 3 nodes. These routes were selected from all possible *triplets* found from a set of nodes randomly placed into several network scenarios with different size and node density. The *triplets* were discovered using the Dijkstra's Shortest Path Algorithm [7]. The shortest route between any pair of the nodes will be formed by the set of intermediate nodes with the minimum number of links (hops). The routes discovered by this procedure are independent from the routing protocols and, in fact, it is expected that an efficient routing protocol will find such routes.

In order to obtain this histogram, we developed a statistical analysis, following this procedure: 1) At time zero, we selected source and destination nodes so the size of the overlapping region (described by the factor h) was constant. 2) A node was randomly placed as forwarding node between source and destination. 3) We assigned random trajectories for the three nodes involved and let the nodes move at a constant speed $v_k = 1\ [m/s]$. 4) We used Equation 23 to calculate the instant when the distance between either source-forwarding nodes or forwarding-destination nodes exceeded R (the transmission range). 5) we repeated the same procedure 100,000 times for multiple positions and trajectories of the three nodes.

This histogram was used to determine the Probability Density Function (PDF) associated to the time that a forwarding node remained inside of the corresponding overlapping zone. This analysis helps to find a mathematical expression that let us calculate the average route duration of a given route formed by 3 nodes, considering all possible initial positions and trajectories followed by every node of the route. We experimentally found a good match between the sample data and the following model:

$$P\left(t\right) = \alpha_1 e^{-\left(\frac{t-\beta_1}{\delta_1}\right)^2} + \alpha_2 e^{-\left(\frac{t-\beta_2}{\delta_2}\right)^2} \tag{24}$$

where, parameters α_i, β_i, δ_i are found using the Robust Non-Linear Least Squares Fitting Method applying it the Trust-Region Algorithm, (see the solid curve in Figure 4). This model considers all possible initial positions of the forwarding node and all possible trajectories, but it does not consider all the possible values of factor h at time zero, when the route is created. Some goodness of fit parameters for this curve fitting method are: $SSE \approx 2 \cdot 10^{-5}$, $R - Square \approx 0.99$, $Adjusted_R - Square \approx 0.99$ and $RMSE \approx 1.5 \cdot 10^{-4}$.

Table 1

Statistical Parameters for PDF shown in Figure 4.

PDF Statistical Parameters						
h [m]	α_1	β_1	δ_1	α_2	β_2	δ_2
$0.28R$	0.008	0.00	85.00	0.0010	225.00	210.00

Table 1 shows the values of the statistical parameters α_i, β_i, δ_i obtained for the initial value of $h = 0.28R\ [m]$.

5 Route Analysis (K-node Mobile Case)

For convenience, we will not use the same notation of nodes we used previously. In this section, we will consider a route formed by K nodes, including source and destination nodes. Each node will be identified by an integer number k, ($k = 0, 1, 2, \cdots, K-1$, where source node is $k = 0$ and destination node is $k = K-1$). Thus, the route will have $N = K-2$ intermediate nodes. We approach this complex problem, dividing the route into N simpler 3-node-mobile routes (*triplets*) which we already analyzed in the previous section. To calculate the time each forwarding node remains inside of its associated overlapping region we need to use Equation 23, but replacing the indexes S, I and D for the corresponding k values of each node of the *triplet*.

This case is even more complex to analyze than previous cases because each overlapping region for each of the N intermediate nodes will have a different size (a different initial h value). To simplify the analysis of routes involving a set of N intermediate nodes, we estimate the average route duration for a route formed by K nodes by taking N samples of a single PDF (defined by Equation 24). The PDF samples generate N random variables, given by $[T'_1, T'_2, ..., T'_N]$. Then, we computed the duration of a route involving K nodes, T'_{R_K}, as:

$$T'_{R_K} = \min\left(T'_1, T'_2, \cdots, T'_n, \cdots, T'_N\right) \tag{25}$$

$$n = 1, 2, \cdots, N \quad ; \quad N = K-2$$

where:

T'_n: are independent random variables, each one representing the time that the intermediate node remains in its overlapping region.

n : identification number of each intermediate node.

N : maximum number of intermediate nodes in the route.

We repeated these computations 1,000 times for different routes with N intermediate nodes and we estimated the average route duration in each case, results are shown in Figure 5 represented by the proposed model curve. As we expected, the time duration of routes decreases as the number of intermediate nodes increases. We should say that another method would be to select a different value of h for each intermediate node and sample its associated PDF. However, this is a more complex method and we found that sampling only one PDF with an average value of $h = 0.28R$ $[m]$ provides a very good precision when we compare our analytical model with simulations, as we can see in Figure 5. It is important to point out that the precision improves as the number of intermediate nodes increases, as we can see in Figure 5. In Section VI we explain the procedures we followed to get our simulation results.

6 Simulation and Results

We performed simulations to validate our analytical model. Several scenarios were created using the NS-2 network simulator [8]. The simulation settings consisted of a $2000\ [m] \times 2000\ [m]$ network with 400 nodes. In this case, we considered the transmission range defined by the IEEE 802.11 standards (i.e., $R = 250\ [m]$). We used the Random Way Point (RWP) mobility model, which is a mobility model commonly used in MANET networks. Briefly, in the RWP model: Each node moves along a line from one point to the next. These points are uniformly distributed over a given convex area, e. g. unit circle. At the start of each trip a random velocity is chosen from a uniform velocity distribution, limited to $v_k \in (v_{k_{\min}}, v_{k_{\max}})$. In our case the velocity was constant $v_k = 1\ [m/s]$. Optionally, the nodes may have pause times when they reach each point before continuing on the next stage, where pause durations are i. i. d. random variables.

We used the simulator without any modification to discover routes involving N intermediate nodes. Then, we let the simulation run until the first intermediate node left the

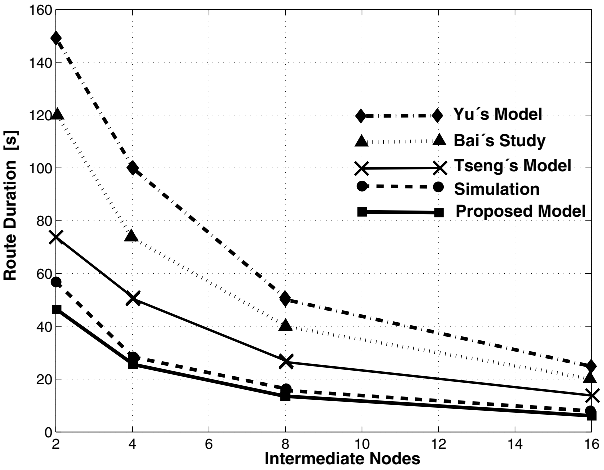

Figure 5. Route duration versus number of intermediate nodes

route and we registered the time interval while the route was available (i.e., $T_R = T_F - T_D$; where: T_R: route duration time, T_F: route failure time, T_D: route discovery time). We performed 1,000 simulations using these routes to obtain sufficient results to validate our analytical model.

From Figure 5, we can see that relative errors between the model and simulation results were found around 20% for routes with 2 intermediate forwarding nodes, whereas relative errors oscillated between 6% and 3% for routes with 4, 8 and 16 intermediate nodes. We consider that the main reason why relative errors were larger for routes with small N is due to the variability of overlapping regions at time zero that we did not consider (we used a single value of h in the model), whereas relative errors for routes with larger values of N are smaller because the average h value for their overlapping regions is closer to the average value we used. It is important to point out that the fact of having a maximum margin of error of 20% may be even acceptable for most of the applications due to the complexity of this problem.

To give more validity to our model and results, we repeated previous simulations but with a higher node density scenario (i.e., $1000\,[m] \times 1000\,[m]$ network with 300 nodes). It is important to note that we obtained consistent results with this simulation scenario within 5% variations with respect to the results in Figure 5.

In addition, Figure 5 compares the results presented in this study with the analytical models presented in [15] and [17]. We selected these studies, because we consider that they addressed the same problem and provided a solution by means of different approaches. Also, they presented analytical expressions and displayed curves relating route duration to route length, so they can be easily compared with our proposed model. Briefly, in [15] the authors presented a graph with the expected values of route duration versus route length and we used these results to compare them with our model and simulations. In [17], the authors provided a graph with normalized values of average route duration time versus number of hops in the route. In this study, the authors indicated that normalized values of average route duration time must be by multiplied by factor (R/v) to adjust them to any specific scenario. A comparison between these analytical studies and our model is presented in Figure 5. Clearly, it shows that our model has a greater precision with respect to simulation results.

Also, in Figure 5 we compare our model versus the empirical study presented in [3]. In [3], the authors proposed an approximated function to estimate route duration (i.e., $T_R \approx \frac{R}{(\lambda_0 N_h v)}$; where: T_R: route duration, R: transmission range, λ_0: experimental parameter (determined by network layout, node density and other parameters of mobility models or scenarios), N_h: number of hops, v: speed) but they did not justify this function with any mathematical validation. A comparison between this experimental study and our model is also shown in Figure 5. It is clear to note that our model has better accuracy with respect to simulation results.

7 Conclusions

In this paper, we proposed a model to estimate route duration in wireless ad-hoc networks when nodes move following random trajectories at constant speeds. This model analyzes a route formed by N intermediate nodes. First, we approached this problem by studying 3-node routes. We showed that route duration of a route formed by N intermediate nodes can be modeled as the minimum route duration of N 3-node routes. Theoretical analyses and simulations were developed to validate this study. In general, simulation results were very close to the results obtained by the proposed model with an acceptable margin of error. Results from this work can be used to compute the signaling overhead of unicast and multicast routing protocols for mobile ad-hoc networks because every time a route fails, the routing protocol needs to either repair the route locally or find a new route. This route duration model can be used to scale up/down the maximum network size as to meet minimum route duration requirements to guarantee a satisfactory communication. In future work we plan to study the behavior of our route duration model with other mobility models and the relationship between route duration and system performance (throughput and overhead), since this has not been established yet.

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References

[1] S. Arbindi, K. Namuduri, and R. Pendse. Statistical Estimation of Route Expiry Times in On-demand Ad-hoc Routing Protocols. *Proc. of the Second IEEE International Conference on Mobile Ad-hoc and Sensor Systems (MASS 05), Washington (USA)*, November 2005.

[2] F. Bai, N. Sadagopan, and A. Helmy. The IMPORTANT Framework for Analyzing the Impact of Mobility on Performance Of RouTing protocols for Ad-hoc NeTworks. *Ad-Hoc Networks Journal - Elsevier Science*, 1(4):383–403, November 2003.

[3] F. Bai, N. Sadagopan, B. Krishnamachari, and A. Helmy. Modelling Path Duration Distributions in MANETs and their Impact on Reactive Routing Protocols. *IEEE Journal on Selected Areas in Communications*, 22(7):1357–1373, September 2004.

[4] C. Bettstetter, G. Resta, and P. Santi. The Node Distribution of the Random WayPoint Mobility Model for Wireless Ad-hoc Networks. *IEEE Transactions on Mobile Computing*, 2(3):257–269, July-September 2003.

[5] A. Campbell, S. Eisenman, and et al. MetroSense Project People-Centric Sensing at Scale. *Workshop on World-Sensor-Web (WSW 2006), Boulder*, October 2006.

[6] S. Cho and J. Hayes. Impact of Mobility on Connection Stability in Ad-hoc Networks. *Proc. of IEEE Communication Society, WCNC 2005, New Orleans (USA)*, 3:1650–1656, 2005.

[7] E. W. Dijkstra. A Note on Two Problems in Connection with Graphs. *Numer. Math.*, 1:269–271, 1959.

[8] K. Fall and K. Varadhan. The NS Manual. *The VINT project, UC Berkeley, LBL, USC/ISI and Xerox PARC*, July 2003.

[9] J. Gomez and A. T. Campbell. Using Variable-Range Transmission Power Control in Wireless Ad-hoc Networks. *IEEE Transactions on Mobile Computing*, 6(1), January 2007.

[10] I. Gruber and H. Li. Link Expiration Times in Mobile Ad-hoc Networks. *In Proc.Workshop on Wireless Local Networks (WLN), in IEEE Local Computer Networks Conference (LCN), Tampa, Florida*, November 2002.

[11] Y. Han, R. La, A. Makowski, and S. Lee. Distribution of Path Durations in Mobile Ad-hoc Networks – Palm's Theorem at Work. *16th ITC Specialist Seminar on Performance Evaluation of Wireless and Mobile Systems, Antwerp, Belgium*, August-September 2004.

[12] X. Hong, M. Gerla, G. Pei, and C.-C. Chiang. A Group Mobility Model for Ad-hoc Wireless Networks. *In Proceedings of ACM/IEEE MSWiM-99, Seattle, WA*, August 1999.

[13] L. Leithold. *The Calculus 7*. Oxford University Press, Oxford University, 2002.

[14] A. Nasipuri, R. Castaneda, and S. R. Das. Performance of Multipath Routing for On-Demand Protocols in Ad-hoc Networks. *ACM/Kluwer Mobile Networks and Applications (MONET) Journal*, 6(4):339–349, 2001.

[15] Y. Tseng, Y. Li, , and Y. Chang. On Route Lifetime in Multihop Mobile Ad-hoc Networks. *IEEE Trans. on Mobile Computing*, 2(4):366–376, October 2003.

[16] D. Turgut, S. Das, and M. Chatterjee. Longevity of Routes in Mobile Ad-hoc Networks. *Proc. of IEEE Vehicular Technology Conference VTC 2001, Rhodes, Greece*, 4:2833–2837, 2001.

[17] D. Yu, H. Li, and I. Gruber. Path Availability in Ad-hoc Networks. *Telecommunications, ICT 10th International Conf.*, 1:383–387, March 2003.